

SOLITONIC AND NON-SOLITONIC Q-STARS*

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Abstract

The properties of several types of Q-stars are studied and compared with their flat space analogues, i.e. Q-balls. The analysis is based on calculating the mass, global U(1) charge and binding energy for families of solutions parametrized by the central value of the scalar field. The two most frequently used Q-star models (differing by their potential term) are studied. Although there are general similarities between both Q-star types, there are important differences as well as new features with respect to the non-gravitating systems. We find non-solitonic solutions which do not have a flat space limit, in the weak (scalar) field region as well as in the opposite region of strong central scalar field for which there does not exist Q-ball solutions at all.

Q-balls [1] are a simple kind of non-topological solitons which occur in a wide variety of (theoretical) physical contexts [2, 3, 4, 5, 6, 7, 8, 9] like the supersymmetric Standard Model [2, 3].

Most of the Q-ball studies are based on the “original” flat space Q-balls. However, it is evident that for a large enough mass scale, gravitational effects become important and one needs to study Q-stars [10] which are their self-gravitating generalizations. That is, they are finite mass and charge solutions of the following U(1) symmetric action:

$$S = \int d^4x \sqrt{|g|} \left(\frac{1}{2} (\nabla_\mu \Phi)^* (\nabla^\mu \Phi) - U(|\Phi|) + \frac{1}{16\pi\mathcal{G}} R \right) \quad (1)$$

where the potential function is usually taken to be:

$$U(|\Phi|) = \frac{m^2}{2} |\Phi|^2 - \frac{\alpha m^{4-p}}{p} |\Phi|^p + \frac{\lambda m^{4-q}}{q} |\Phi|^q. \quad (2)$$

Two choices are popular in the literature: $p = 3, q = 4$ and $p = 4, q = 6$. Actually, this system allows for a different kind of localized solutions already without self-interaction (i.e. only mass term) or with an additional $|\Phi|^4$ term, namely, boson stars [11, 12, 13]. Boson stars have also a conserved global U(1) charge, but unlike Q-stars, they do not have a flat space limit.

I will assume spherically-symmetric solutions with non-vanishing U(1) charge, i.e. $\Phi = mf(r)e^{i\omega t}$ and $ds^2 = A^2(r)dt^2 - B^2(r)dr^2 - r^2(d\theta^2 + \sin^2\theta d\varphi^2)$ so the charge and mass are given by

$$Q = 4\pi\omega m^2 \int_0^\infty dr r^2 (B/A) f^2, \quad M = 4\pi \int_0^\infty dr r^2 \left[\frac{\omega^2 m^2 f^2}{2A^2} + \frac{m^2 f'^2}{2B^2} + U(f) \right]. \quad (3)$$

Without loss of generality we can assume $\omega > 0$ so we will have $Q > 0$ as well.

The existence of Q-stars was demonstrated by Friedberg *et al* [14] and by Lynn [10] together with a presentation of the basic properties of the solutions for the 2-4-6 potential. A discussion of 2-3-4 Q-stars appeared only quite recently [15]. It was shortly followed by a study [16] which showed that gravity limits the size of Q-balls. On the other hand, a recent analysis [17, 18] of spinning Q-balls and Q-stars is concentrated in the 2-4-6 case.

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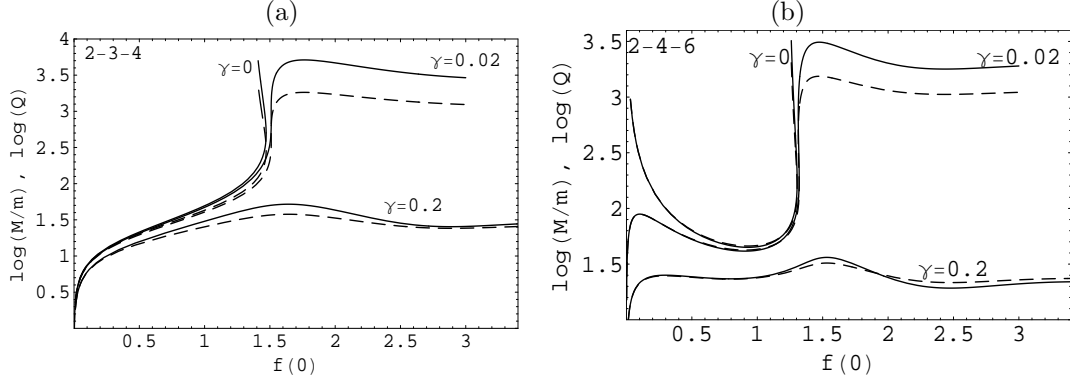


Figure 1: Plots of $\log(Q)$ (solid line) and $\log(M/m)$ (dashed) vs. $f(0)$ for $\gamma = 0$ (Q-balls), $\gamma = 0.02$ and $\gamma = 0.2$. (a) 2-3-4 Q-stars; (b) 2-4-6 Q-stars.

I give here the main results of a systematic comparative study of both kinds of Q-stars, including the dependence on the gravitational strength parametrized by the dimensionless parameter $\gamma = 4\pi G m^2$. A more detailed summary will be presented elsewhere [19]. I choose in both cases the parameters $\alpha = 2$ and $\lambda = 1$ so the potentials will have a similar form. The gravitational parameter γ will take the following three values: $\gamma = 0, 0.02, 0.2$.

Figure 1 summarizes the main results in the $Q - f(0)$ and $M - f(0)$ planes namely, the general behavior of the charge and mass. Figure 2 depicts the binding energy per particle (in dimensionless form), $1 - M/mQ$ as a “function” of Q which is more instructive from a physical point of view. It is evident from this figure that Q-stars are more strongly-bound than their non-gravitating counterparts with the same Q . From our results one can draw the following observations and conclusions:

Already in flat space there is a very significant difference between the “thick wall” (small $f(0)$) solutions of the two potentials: the thick wall Q-balls of the 2-3-4 potential are small and stable, i.e. Q and M vanish as $\omega \rightarrow m$ or $f(0) \rightarrow 0$ while $M/m < Q$. On the other hand, those of the 2-4-6 potential are large and unstable, i.e. both charge and mass diverge while $M/m > Q$ in the same limit.

Gravity introduces significant changes such as allowing solutions in regions where flat space solutions do not exist and limiting the charge and mass of Q-stars. But still the changes are quite small for weak scalar field 2-3-4 Q-stars, as seen in figures 1a and 2a. On the other hand, gravity changes completely the nature of the weak field 2-4-6 solutions even for a small γ (say, 0.02) as figure 1b shows: as $f(0) \rightarrow 0$ the charge and mass do not blow up, but on the contrary go to zero. Looking from the other direction, one sees that the charge and mass start rising from zero, reach a local maximum, decrease a little and

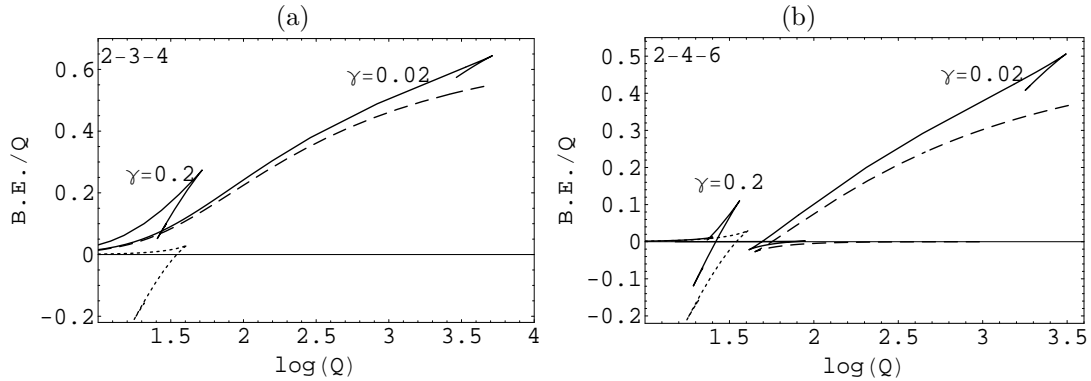


Figure 2: Plots of binding energy per particle $(mQ - M)/mQ$ vs. $\log(Q)$ for $\gamma = 0$ (dashed), $\gamma = 0.02$ and $\gamma = 0.2$ and for boson stars with $\gamma = 0.2$ (dotted). (a) 2-3-4 Q-stars; (b) 2-4-6 Q-stars.

then go to the thin wall region and beyond as described below. Moreover, unlike the 2-4-6 Q-balls for $f(0) \ll 1$ which were unstable, now there appears a region of stability (below the resolution of figures 1b or 2b) for small enough $f(0)$. This is followed by a region of unstable solutions up to a certain value of $f(0)$. From this point on, all solutions are stable.

For larger values of $f(0)$ we encounter for both potentials “thin wall” Q-stars quite similar to the corresponding Q-balls, although the self-gravitating solutions are not so well described by the thin wall approximation. The reason is that where the thin wall approximation in flat space is accurate, gravity already causes deviations. To push it to the extreme, the thin wall approximation becomes exact for $Q \rightarrow \infty$, but gravity keeps Q-stars away from this best region by introducing a maximal value of Q . For large values of γ there are no thin wall solutions altogether.

Another new gravitational effect is the existence of solutions when the central field becomes considerably larger than $f_*(0)$ which is the flat space critical field. Unlike the Q-ball case, the mass and charge curves cross this point and there are solutions as far as I was able to explore numerically. All small γ solutions are stable, but their nature for $f(0) \gtrsim f_*(0)$ becomes quite different from the Q-balls as we go further. Moreover, it is obvious that this kind of solutions cannot be considered solitonic as they do not have a flat space limit: while the charge and mass of the solutions with $f(0) < f_*(0)$ have a (finite) limit as $\gamma \rightarrow 0$, those in the other region blow up. In other words, it is only thanks to gravity that this kind of solutions with $f(0) \gtrsim f_*(0)$ exists.

References

- [1] S. R. Coleman, Nucl. Phys. B **262**, 263 (1985) [Erratum-ibid. B **269**, 744 (1986)].
- [2] A. Kusenko, Phys. Lett. B **405**, 108 (1997)
- [3] K. Enqvist and J. McDonald, Phys. Lett. B **425**, 309 (1998)
- [4] A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B **418**, 46 (1998)
- [5] K. Enqvist and J. McDonald, Nucl. Phys. B **538**, 321 (1999)
- [6] A. Kusenko, Phys. Lett. B **404**, 285 (1997)
- [7] G. R. Dvali, A. Kusenko and M. E. Shaposhnikov, Phys. Lett. B **417**, 99 (1998)
- [8] K. Enqvist and A. Mazumdar, Phys. Rept. **380**, 99 (2003)
- [9] M. Dine and A. Kusenko, Rev. Mod. Phys. **76**, 1 (2004)
- [10] B. W. Lynn, Nucl. Phys. B **321**, 465 (1989).
- [11] P. Jetzer, Phys. Rept. **220**, 163 (1992).
- [12] T. D. Lee and Y. Pang, Phys. Rept. **221**, 251 (1992).
- [13] A. R. Liddle and M. S. Madsen, Int. J. Mod. Phys. D **1**, 101 (1992).
- [14] R. Friedberg, T. D. Lee and Y. Pang, Phys. Rev. D **35**, 3658 (1987)
- [15] A. Prikas, Phys. Rev. D **66**, 025023 (2002)
- [16] T. Multamaki and I. Vilja, Phys. Lett. B **542**, 137 (2002).
- [17] M. S. Volkov and E. Wohnert, Phys. Rev. D **67**, 105006 (2003)
- [18] B. Kleihaus, J. Kunz and M. List, Phys. Rev. D **72**, 064002 (2005).
- [19] Y. Verbin, *in preparation*.